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The role of Fintech Firms in intermediating the trading of stocks during Covid-19

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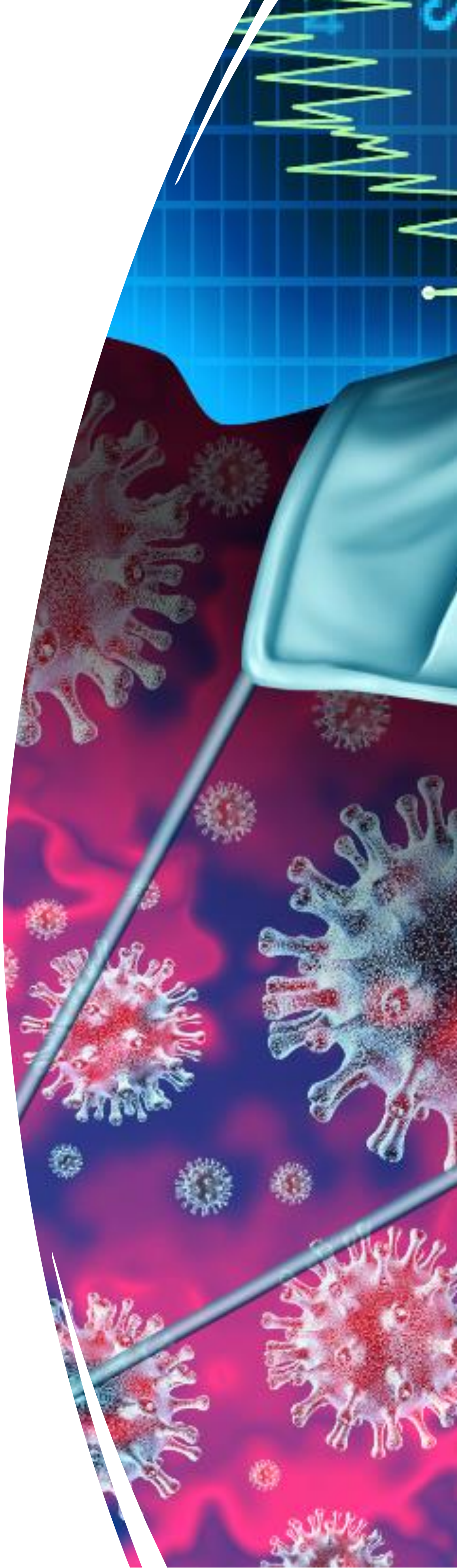

The role of fintech firms in intermediating the trading of stocks during Covid-19

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Abstract

The participation of retail investors in the online trading of stock market surged during the year 2020, with an even seemingly large set of new investors starting to trade stocks on fintech platforms for the first time. This development could seem surprising, as the Covid-19 pandemic has likely increased uncertainty and entailed negative wealth effects. In most canonical models of stock trading, at least one of these effects would imply a reduced demand for risky assets (such as stocks). This paper develops a model which incorporates both effects and maintains the assumption of weak form efficient markets. It shows that the observed surge of demand is best explained by there being investors whose trading is based on common sentiment analysis rather than fundamental analysis. Reduced opportunity costs of participation can help further. We provide arguments that both trends have increased over the past year. The paper also contributes to the REE literature by considering wealth effects and sentiment effects jointly in a stylized setting that has an analytical solution. It derives new predictions on the relationship between stock market participation and asset prices.

JEL classification: G12, G14, G40

Keywords: Covid-19, efficient markets, rational expectation equilibrium, sentiment, stock market participation, wealth effects

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1 Introduction

Covid-19 was identified by the World Health Organization as a global pandemic risk in early 2020, and by March 2020, infection and death statistics had got out of hand worldwide, so that most countries decided to implement measures of social distancing, encouraging working and studying from home. It was then feared that many sectors (retail, restaurants and hospitality services, etc) would experience a contraction. As immediate reaction to the fear of a global supply chain shock, many stock prices and indexes fell rapidly around the same period of time; for example, NASDAQ Composite fell from about 9700 in mid February 2020 to about 6900 around late March 2020 (about 40% loss), and similar patterns were echoed in other stock indexes: e.g., Dow Jones retail Index (USA), FTSE (UK), DAX (Germany), S&P/TSX (Canada) all experienced sudden losses of value in the magnitude of 40-50% during the same interval.

At the time, it was not obvious whether this stock market movement would persist or not. On the one hand investors might have believed that the fall in stock prices was too large, and bound to be temporary - because firm values largely reflect cash flows that will also materialize over several periods of time after the Covid-19 crisis is over. On the other hand, there was an increased risk that many firms would not survive and go out of business. Both of these effects increase uncertainty about asset returns, and may have also lead to an unusually large wedge between investors' beliefs and stock prices.

Relatedly, the Covid-19 situation posed an interesting question regarding stock market participation, especially when we consider the participation of retail investors. One hypothesis was that the participation in stock market would fall because individuals faced negative wealth and liquidity shocks. Another hypothesis was that some previously not participating investors would join the stock market, because of the extra time they had to overcome the fixed cost of participation, and/or because they might expect high returns, be it for rational or behavioral reasons. The patterns of new participants are arguably most visible when we focus on FinTech platforms that bring greater convenience in establishing an investment account for online trading. Well known examples of such platforms are Robinhood in the USA and Wealthsimple in Canada.

In this paper, we will provide a brief overview of platforms that offer online stock trading in Canada, and highlight stylized facts about the dynamics of participation in these platforms as well as in their USA equivalents during the period of interest. There appears to be evidence that these platforms have onboarded large number of new clients during the period. As it would be a stretch to argue that investors have become wealthier or less risk averse, it calls for a theoretical explanation based on subjective expectations of high returns, together with a possible greater willingness to pay for the non-monetary fixed costs needed for participation.

Among the stylized facts presented, we highlight the indications of prices being driven by retail investors demanding assets for non-fundamental reasons. The most famous incidence in this spirit is the GameStop Corp share price that experienced an unprecedented price increase and volatility in early 2021, which was arguably heavily affected by discussions in social media and by Elon Musk's tweet of Jan 26, 2021, cryptically stating "Gamestonk!!". We provide other examples in this spirit.

To interpret these stylized facts in the context of the aforementioned hypotheses, we then develop an illustrative model of retail investors' stock market participation incentives and their price impact. For the sake of clarity, there are just two assets: a risky asset (e.g., a stock or a portfolio of stocks), and a risk-free asset (e.g., savings in a bank or a portfolio of risk-free bonds). In the model, there are three types of investors. First, there are sophisticated informed investors who trade based on their superior knowledge about the fundamental value of the risky asset (e.g., professional investment and hedge funds). Incorporating standard assumptions regarding such traders in the REE (rational expectations equilibrium) literature, the demand of these traders does not depend on wealth. Second, there are non-sophisticated retail investors, who first choose whether to participate in the stock market, and then trade based on their common subjective beliefs about the risky asset value. Retail investors are heterogenous in wealth and their risk tolerance is endogenous - a higher wealth makes investors more risk tolerant/less risk averse. This implies that there is a threshold level of wealth below which investors choose not to participate. Third, there are (risk-neutral) competitive market makers who do not know the fundamental value and the beliefs of retail investors, but learn noisy information about the fundamental value from prices and order flows. Market makers implement the market efficiency condition by setting the asset price to be equal to the expected value of the fundamental based on all public information.

The model shows that in the context of the Covid-19 shock, where wealth distribution is unlikely to have had a positive mean shift, the wedge between the beliefs of retail investors about risky asset values and the price on the one hand, and the non-monetary aspects of participation costs on the other hand, may be the more plausible explanations for the surge in the demand of retail investors. The analysis of the implied equilibrium price patterns highlights further interesting facts. For example, the increased demand by retail investors, trading on the basis of non-fundamentals signals, monotonically reduces the importance of fundamental information as a driver of equilibrium asset prices. At the same time, it increases the weight on prior beliefs about fundamental values and generates non-monotonic price effects regarding the role of retail investors' common signals. At the limit and in our baseline model, retails investors' signals are uncorrelated with the fundamental, and the model highlights intuitive parameters under which price distortion is maximized. As the assumption that retail investors' demand is always driven by non-informative signals may seem a bit too radical, we also provide an extension where some new retail investors have fundamental information. As one would expect, all the main effects remain, but are smaller in their magnitude.

From a theoretical modelling perspective, our paper builds on methods from the large literature of REE models (see e.g., seminal papers by Grossman and Stiglitz (1980, 1988), Brown and Jennings (1989), Vives (1995), Kyle (1985) and Glosten and Milgrom (1985), and many models that build on these settings, see Brunnermeier (2001) and Vives (2008) for surveys). Our formal solution method is most similar to the one in Vives (1995), where the asset price is set to implement the (weak form) market efficiency condition (as in Kyle (1985)), while considering traders that operate in a competitive market and are risk averse. Many classical settings in the REE literature abstract from considerations of wealth effects for the benefit of analytical solutions (by adopting the CARA assumption). At the same time, there is overwhelming evidence that wealthier investors are less risk averse and more likely to participate in the stock market. To capture such a realistic wealth effect, we incorporate the modelling method in Peress (2004).[1]

Our assumption that retail investors' trading incentives are primarily driven by considerations outside the superior knowledge of the fundamental value, is consistent with many models and empirical evidence in the literature focused on behavioral finance (see e.g. Barber and Odean (2000, 2001), Kahneman and Tversky (1980, 2013)). These arguments also point towards retail investors tending to have losses on average when competing against informed traders and rational market makers. There is also empirical evidence that the possibility to trade on online platforms encourages retail traders to take excessive risks and buy "lottery-like" stocks[2] (see e.g., Kumar (2009), Kalda (2021)) and to participate more (Choi et. al. (2002), Bogan (2008)). Our baseline modelling approach regarding retail investors' information is closest to Mendel and Shleifer (2012), where retail investors are "noise traders" with correlated beliefs.

The existing empirical and theoretical literature on Covid-19's impact on stock trading is still at its early stages. There are some notable empirical findings, which tend to be consistent with our findings. For example, van der Beck and Jaunin (2021) confirm the insight that retail investors' demand soared during the Covid-19 pandemic. By focusing on the Robinhood Market Inc. platform, they find large price impacts driven by retail investors. The paper by Glossner et. al. (2021) documents that institutional investors experienced client outflows, and platforms

[1] In such setting, each individual investor's utility function is still CARA. However, different investors have different risk tolerance parameters, which depend on their wealth. While there are many other utility functions (the family of homothetic preferences, including CRRA) that produce similar aggregate wealth effects in models with representative agents, this modelling choice enables an informative analytical solution. For the application of such utility functions in the context of micro founded trading models, see e.g. Kyle and Xiong (2001).

[2] Stocks that have low expected value compared to the price, high variance and positive skewness, based on public data.

like Robinhood's clients acted as liquidity providers. While there are other papers confirming such key empirical patterns, the set of theoretical models and interpretations is more scarce.

Finally, our paper relates to the broader macroeconomic literature on stock market participation. For example, Mankiw and Zeldes (1991) find that the percentage of U.S households with direct holdings of at least $1000 in the stock market is only 23.2%, Bertaut and Starr-McCluer (2000) find that in the US less than 50% of households own some form of stock. Based on Giannetti and Koskinen (2010) Canadian investors' participation rate in the domestic stock markets is 25%, and foreign equity held by domestic investors is 30.2%. While the focus of this paper is a partial equilibrium one, it is plausible that over the long term there could be some positive effects of engaging new investors, who may become more sophisticated over time, which may bring some positive macroeconomic effects.

2 Online trading platforms and stylized facts

2.1 Players in the Canadian Market

All the big five banks that dominate the banking industry in Canada, BMO, Scotiabank, CIBC, RBC and TD, offer self-directed trading platforms. These banks offer both mobile and desktop trading while requiring varying levels of account minimum and charging varying levels of commission per equity trade. Table B.1 in the Appendix details how these platforms differ in terms of charges, app store ratings among other things. Typically, the pricing structure offered by these banks is dependent on the frequency of trades, where a lower flat fee is charged for clients with a higher number of trades per quarter.

In addition to these traditional banks, there exist brokerage firms, like Interactive Brokers (IB), that operate in the Canadian market. IB's pricing structure is different from those offered by traditional brick and mortar banks as it offers a fixed and tiered pricing structure. The tiered pricing structure sets up varying fee levels dependent on the monthly volume of shares; it further imposes a minimum and a maximum commission per order[3] Whereas the fixed pricing structure imposes a flat fee along with a maximum and a minimum cap on commission per order. IB proposes more sophisticated trading platforms and tools that are targeted towards professional traders, while being largely accessible to retail investors at the same time.

In addition to Interactive Brokers, there exist firms that specifically provide online brokerage services, such as Questrade, Qtrade Investor, Virtual Brokers, but who have managed to set themselves apart from traditional financial institutions. For instance, commission free trading was first offered in Canada in 2009 by Virtual Brokers.[4]

[3] See IB - Pricing structure at https://www.interactivebrokers.ca/en/index.php?f=45251&p=stocks2

[4] Goldman, Andrew 2021, "Best Trading Platforms in Canada - 2021 Guide" Wealthsimple,

Finally, Wealthsimple Trade is the online trading app established by Wealthsimple Inc. an investment management service that was founded in 2014. Wealthsimple trade is a mobile-only platform whose specificity is to offer unlimited commission free trades along with no account minima.

These mobile-only platforms typically tend to have more easy-to-use interfaces and garner higher ratings and positive reviews on the Apple app store. Some of these platforms, such as Wealthsimple and Questrade, even offer to their clients services that go beyond what is referred to as "self-directed investing", where clients can buy and sell various investments (stocks, ETFS, etc), and the option to invest in managed portfolios.

It should be noted, however, that professional traders do not seem to favor these platforms, possibly due to delays in price quotes, or changes in exchange rates that can impact their potential gains.[5]

2.2 Observed surge in online stock trading by retail investors

In 2020, many trading platforms experienced a sharp increase in the number of trading accounts. There is strong evidence supporting increased user registration and activity across trading platforms operating in the United States and Canada. For example, Etrade Financial Corporation, a popular American electronic trading platform, saw a record 260,493 accounts open in March, 2020, which is a sharp increase from the numbers observed in the past [6] In Canada, Wealthsimple Trade saw an increase of its user base by 80%, reaching 380,000, between July and December 2020.[7] Interactive Brokers reported $523 million in revenues Q2 2020, to be contrasted with $488 million in Q2 2019, and which was attributed to a strong growth in commission revenue (+ 55% from Q2 2019 to Q2 2020). [8]

Figure 1 provides a different indication of increased interest in stock trading since Covid-19 was identified as a pandemic. It documents Google search trends data over the past 5 years. The left panel plots the search interest in stock trading in general, and the right panel provides an arguably more accurate proxy of the interest from unexperienced investors. Both figures clearly indicate a peak of interest after the mid-March of 2020, and another one in 2021

https://www.wealthsimple.com/en-ca/learn/best-trading-platforms-canada

[5]See e.g., reviews at the App Store, https://apps.apple.com/

[6]Wursthorn, Michaelm Mischa Frankl-Duval and Gregory Zuckerman 2020, "Everyone's a Day Trader Now", The Wall Street Journal July 25, 2020. https://www.wsj.com/articles/everyones-a-day-trader-now-11595649609

[7]Balakrishnan, Anita 2020, "As thousands of Canadians flock to day trading during lockdown, experts urge caution: A growing number of investors are trying their hand at day trading", December 17, 2020, IE|Investment Executive, https://www.investmentexecutive.com/news/research-and-markets/active-trading-more-popular-among-canadians-in-2020/

[8]Businesswire 2020 "Interactive Brokers Group Announces 2Q2020 Results" July 21, 2020 https://www.businesswire.com/news/home/20200721005909/en/Interactive-Brokers-Group-Announces-2Q2020-Results

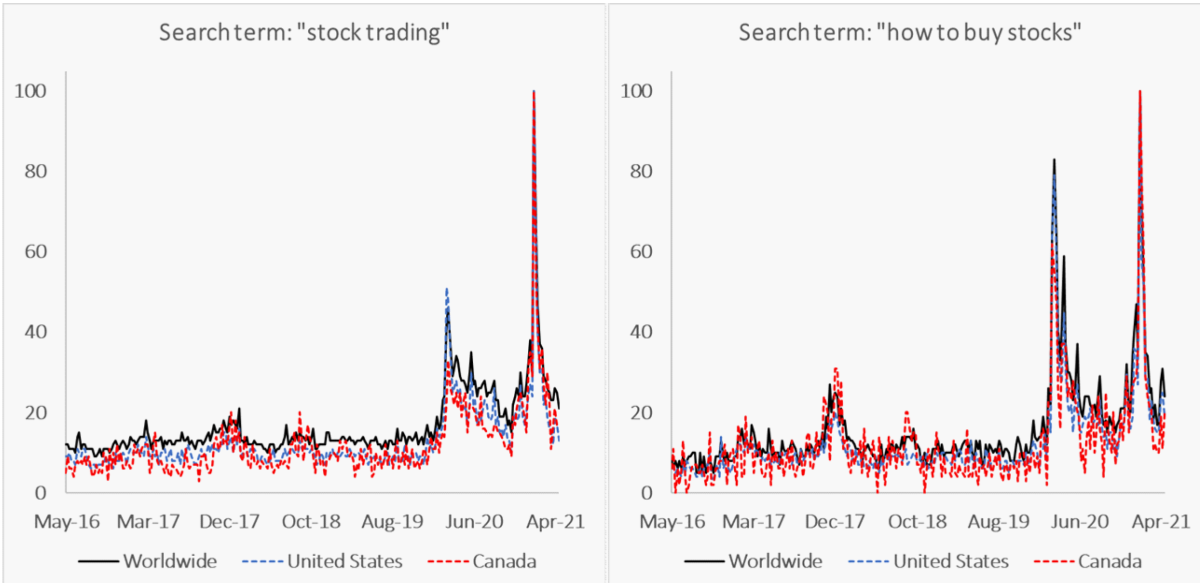

Figure 1: **Google trends**. On both panels, the number of searches in the week with the highest number of searches is normalized to 100, and other values reflect interest relative to that benchmark. Source: https://trends.google.com/

(possibly associated with the widespread coverage on the unusual price patterns of GameStop Corp stock).On average, interest in stocks trading seems to be noticeably higher during the year from the spring 2020 to spring 2021 compared to the previous years. The patterns in Canada, United States and Worldwide are similar.

2.3 Possible drivers for the observed surge in online trading

2.3.1 A lowering of the barriers to entry

Platforms such as Wealthsimple Trade in Canada and Robinhood in the US have minimized several barriers to entry to the trading market. The easy-to-use interface, combined with commission-free trading, no account minima, and with the minimum requirement of having a bank account, in practice enables any 18 and above adult to engage with this platform and become an online investor. More than half of the customers of Robinhood, a popular commission-free trading and investing app based in the US, opened a brokerage account for the first time ever during the period.[9]

[9] Rooney, Kate 2020, "Fintech app Robinhood is driving a retail trading renaissance during the stock market's wild ride", CNBC, June 17 2020. https://www.cnbc.com/2020/06/17/robinhood-drives-retail-trading-renaissance-during-markets-wild-ride.html

2.3.2 Stimulus packages and changes in the consumption/investment pattern

Some commentators have considered that one of the implications of the lockdown was that there were fewer opportunities to buy goods, while there was more time to manage one's finances online.It was also noted that, during the pandemic, many students found themselves eligible for CERB payments in Canada, seeing an increase of income of nearly 2k each month for 7 months, and that some of these students used the entirety of the CERB payment (14k) to invest in a small number of individual stocks.

Similar trends were present in the United States. According to a CNBC article, Americans who received a stimulus check during the pandemic increased their spending by 81% and some of the spending was directed towards buying stocks [10][11] Across various income brackets, securities trading seems to have been one of the expenditures that saw an increase after the disbursement of the stimulus check (As seen in Figure, 2 in that article).

The overall macro impact is however difficult to establish, as in many cases, the stimulus only partially offset a loss of income during the lockdown and the contraction of economic activity. Given the overall slowdown of the economy, it is more likely that the overall effects of Covid-19 on the wealth distribution were not favourable. At the same time this evidence points towards reduced opportunity costs of participation in the stock market and we will incorporate this aspect to our model in Section 3.

2.3.3 Prevalence of sentiment analysis over fundamental analysis

Stock prices in the Covid period have shown significant movement in response to positive or negative signals on social media platforms; for example, the stock prices of several companies like Etsy Inc, Signal Advance Inc and GameStop Corp. have responded strongly to Elon Musk's tweets. Notable examples of these tweets did not contain more than a hint to the company's name. Nevertheless, as shown in the article in Bloomberg BusinessWeek (2021) these tweets led to an immediate upward jump in the mentioned firm's share price.[12] Figure 2 highlights the timing (dotted red line) and content of these tweets along with the mentioned firms' share price dynamics (solid black line) 30 days before and after Elon Musk posted the tweets. It shows that the share price impact of these tweets was rather persistent and lasted at least several days or weeks. This seems to illustrate an increasing reliance on sentiment analysis vs fundamental

[10] Fitzgerald, Maggie 2020, "Many Americans used part of their coronavirus stimulus check to trade stocks", CNBC May 21 2020. https://www.cnbc.com/2020/05/21/many-americans-used-part-of-their-coronavirus-stimulus-check-to-trade-stocks

[11] See also Wursthorn et. at 2020 article in the Wall Street Journal, cited above.

[12] Gambrell, Dorothy 2020 "A Brief History of Elon Musk's Recent Market-Moving Tweets", Bloomberg BusinessWeek, February 11, 2021. https://www.bloomberg.com/news/articles/2021-02-11/how-elon-musk-s-tweets-moved-gamestop-gme-bitcoin-dogecoin-and-other-stocks

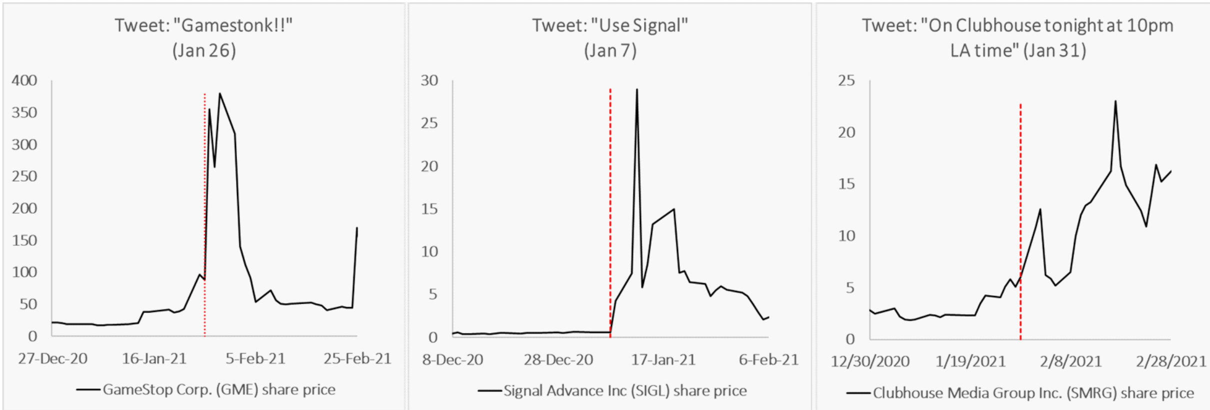

Figure 2: **Examples of share price impact of Elon Musk's tweets in 2021.** These figures is based on daily data of opening prices. The date of the tweet is marked with red dotted lines.

analysis as a driver of investors' demand and prices.

2.3.4 New communities/fora and the democratization of investment advice

As more novice traders enter the market, they rely on community support to move along the learning curve. There has been a lot of mobilization and social buzz on different platforms like reddit and discord. This allows would-be investors to take confidence and to learn more easily about what used to constitute complex technical issues reserved for the specialists.

2.3.5 New forms of coordinated action via social media

The retail trending frenzy that was seen in early 2021 has had considerable spillovers in other regions of the world. FreeTrade, a prominent British trading platform, reported an increase of nearly 160,000 users in January 2021 which was accredited to the increased interest in the "reddit-rally stocks". Their daily onboard rate for new users during this "retail trading boom" nearly tripled [13]

The GameStop Corp short squeeze that was fuelled by a social media mobilization put the spotlight on stock trading and stock trading platforms. As novice traders began sharing on social media the "gains" they had made during this short squeeze on different social media platforms, and as more media coverage on this issue was brought to attention, more retail investors were tempted into trying stock trading for the first time, on platforms like Robinhood, to buy stocks like AMC (AMC Entertainment Holdings Inc) and GME (GameStop Corp).

[13] White, Lawrence and Anna Irrera, 2021, "Reddit-fuelled retail trading frenzy spreads to Europe", Reuters, February 9, 2021. https://www.reuters.com/article/us-retail-trading-europe-idUSKBN2A9284

2.3.6 The millenial investor (passion for technology firms, gaming, cryptocurrencies)

With the increased ease of navigation and accessibility, trading apps such as Robinhood and Wealthsimple have been successful in attracting and targeting young individuals/ millennials into the trading scene (Robinhood has reported its median age of customers as being 31)[14] The sharp increase in the number of novice traders across the board can be attributed to the entry of young people into the online trading landscape. The spike in users was particularly sharp during the first quarter of 2020 where the stock market experienced a downturn and a subsequent recovery; according to some equity strategists, these accounts reflect the entry of new investors who perceived this downturn to be a "generational- buying moment" while having limited knowledge of the equity space [15] The interface built by Robinhood and to some extent Wealthsimple Trade is often compared to that of a social media app or a mobile game. Wealthsimple Trade offers its customers 25$ when a friend signs up and trades at least 100$. Robinhood, in particular, has been remarked to have a gamified easy-to-use interface that further attracts novice investors.

As previously remarked, platforms like Robinhood have led to the gamification of investing and trading to a considerable extent.[16] This gamification can explain the investing activity typically found on this platform. According to the New York Times, it was found that in the first quarter of 2020 Robinhood users traded 40 times as many shares as Charles Schwab (a more traditional brokerage) customers. Gamification comes with the risk of large losses as it may encourage retail investors to pursue trading strategies that are systematically disconnected from fundamental drivers of share prices (e.g., too frequent stock trading is rarely a profitable strategy for retail investors), and instances of large individual losses due to gamification have indeed been documented.[17]

[14] See Rooney 2020 acticle in CNBC, cited above.

[15] Fitzgerald, Maggie 2020, "Young investors pile into stocks, seeing 'generational-buying moment' instead of risk", CNBC, May 12 2020. https://www.cnbc.com/2020/05/12/young-investors-pile-into-stocks-seeing-generational-buying-moment-instead-of-risk.html

[16] Massa, Annie and Edward Robinson, 2020, "Robinhood's Role in the 'Gamification' of Investing, Bloomberg Wealth, December 19 2020. https://www.bloomberg.com/news/articles/2020-12-19/robinhood-s-role-in-the-gamification-of-investing-quicktake

[17] Popper, Nathaniel 2020, "Robinhood Has Lured Young Traders, Sometimes With Devastating Results", The New York Times, July 8, 2020. https://www.nytimes.com/2020/07/08/technology/robinhood-risky-trading.html

3 The model

3.1 The setting

There are two assets: a risk-free asset, which offers certain return normalized to one, and a risky asset with an uncertain fundamental value

$$v = \theta + u, \tag{1}$$

where θ is the explainable part of the fundamental value, and u is the unexplainable part of the fundamental value. Both θ and u are Normally distributed and independent: $\theta \sim \mathcal{N}\left(\theta_0, \frac{1}{\beta_\theta}\right)$, $u \sim \mathcal{N}\left(0, \frac{1}{\beta_u}\right)$. These distributions are common knowledge.

Three types of investors trade these assets: 1) a mass I of sophisticated informed investors, 2) a mass N of retail investors and 3) competitive market makers. At date 0 retail investors choose whether to participate, at date 1 all investors trade, and at date 2 the fundamental value v is realized and investors consume.

Sophisticated informed investors (e.g., institutional investors) are a homogenous set of investors who each observe the value of θ, and the same CARA utility with risk tolerance parameter τ (inverse of risk aversion), and always participate. Each of them chooses demand schedule d, to maximize as the function

$$\mathbb{E}\left[-\exp\left(-\frac{d\,(v-p)+w_I}{\tau}\right)|\theta\right], \tag{2}$$

where w_I is the informed investor's wealth.[18] When choosing their optimal demand schedule, investor know the equilibrium asset price p.[19] The aggregate demand of informed investors is $D = I\int_i \mathbf{1}_i h_i di$.

Retail investors are heterogenous in their wealth, and the wealth distribution is $G(w_i)$ with support $[w_m, \theta)$, where $w_m > 0$, and w_i is independent of θ and u. At date 0 each retail investor i chooses whether to participate in the stock market or not, and his/her choice to participate is denoted with $\mathbf{1}_i \in \{0, 1\}$. If a retail investor chooses to participate, he/she must pay a fixed participation cost F in monetary equivalent units. This cost reflects monetary costs such as the fees of establishing an account, but also non-monetary costs like spending time to find out more about trading and the risky asset. As in Peress (2004), each consumer's risk tolerance parameter $\gamma(w_i)$ is increasing in the consumer's wealth, $\gamma'(w_i) > 0$. As in Mendel and Shleifer

[18]The assumption that w_I is the same across all informed investors is inconsequential because their demand does not depend on w_I. This is a well known feature of CARA utility and their demand will be specified shortly.

[19]This can be interpreted as the investor observing the market price p and submitting a market order in a competitive market, or him submitting a set of limit orders for each price.

(2012) retail investors have biased beliefs about the explainable component of the fundamental value, and do not learn fundamental information from prices. Namely, they consider that the fundamental value of the risky asset is

$$S+u,$$

where S is Normally distributed, $S\sim\mathcal{N}\left(S_0,\frac{1}{\beta_S}\right)$ and u is the unexplainable component as in (1), and S is independent of u, θ and w_i. Retail investors observe the value S if they choose to participate in the stock market, and participating retail investors observe the same value of S. The value S can be viewed as the "sentiment", and may for example be driven by non-fundamentals based coordination in social media platforms.[20]

At date 1, each retail investor i who has decided to participate in the stock market observes the sentiment value S, the risky asset price p, and chooses his/her demand schedule h_i to maximize

$$U_i\left(\mathbf{1}_i=1\right)\equiv\mathbb{E}\left[-\exp\left(-\frac{h_i\left(S+u-p\right)+w_i-F}{\gamma\left(w_i\right)}\right)|S,w_i\right]. \tag{3}$$

The utility of a retail investor who does not participate in the stock market is certain and given by

$$U_i\left(\mathbf{1}_i=0\right)=-\exp\left(-\frac{w_i}{\gamma\left(w_i\right)}\right).$$

Anticipating her/her expected utility of at date 1, each investor i chooses to participate, if and only if,

$$\mathbb{E}\left[U_i\left(\mathbf{1}_i=1\right)\right]\geq U_i\left(\mathbf{1}_i=0\right).$$

When retail investors decide whether to participate, they know the price but do not know the realization of S. The aggregate demand of retail investors is $H=N\int_i\mathbf{1}_ih_idi$.

Finally, there are competitive market makers who implement the market efficiency condition, as they set the price equal the expected fundamental value of the risky asset based on all public information. Namely, market makers do not observe the random variables θ, S, and u, but observe the total order flow $D+H$, which they use to update their beliefs on the fundamental value of the asset, i.e.,

$$p=\mathbb{E}\left[v|D+H\right].$$

Market makers also know the structure of the model and distributions from which random variables are drawn.[21]

[20] An example of this would be the coordination retail trading regarding the GameStop share. This approach also provides one possible way to endogenize noise traders demand. A model where retail investors observe slightly different signals would deliver qualitatively similar results and would needlessly complicate the model. In Section 4 we will also analyze an extension where where some retail investors trade based on fundamental information.

[21] The solution method used in this paper is based on Vives (1995), which considers competitive traders.

3.2 Retail investors' participation

Before analyzing the full problem, let us focus on the drivers of retail investors' participation. For that, we need to first derive the retail investor's optimal demand and utility, should he/she participate. To shorten the notation, denote $c_i \equiv h_i (S + u - p) + w_i - F$. Using the observation that the conditional distribution $c_i|S \sim \mathcal{N}(\mathbb{E}[c_i|S], Var[c_i|S])$, and the functional form of Normal distribution, we can simplify (3) to obtain

$$U_i(\mathbf{1}_i = 1) = -\exp\left(-\frac{1}{\gamma(w_i)}\left(\mathbb{E}[c_i|S] - \frac{1}{2\gamma(w_i)} Var[c_i|S]\right)\right), \quad (4)$$

where $\mathbb{E}[c_i|S] = h_i (S - p) + w_i - F$ and $Var[c_i|S] = h_i^2 \frac{1}{\beta_u}$. From there, it follows that the retail investor's demand has a familiar form

$$h_i = \gamma(w_i) \beta_u (S - p). \quad (5)$$

The optimal demand is higher if the difference between the investors' subjective beliefs about the fundamental value are higher and when there is less uncertainty about the unexplainable components of the fundamental value (higher β_u). Furthermore, the demand is higher when investors are more risk tolerant, and as we assume that risk tolerance is increasing with wealth, wealthier investors' demand for risky asset is higher.

We can then derive the participation condition

Lemma 1 *There exists a threshold level of wealth $\bar{w}$ such that all retail investors with $w_i \geq \bar{w}$ participate in trading the risky asset and all retail investors with $w_i < \bar{w}$ do not. The threshold*

$$\bar{w} = \gamma^{-1}\left(\frac{F}{\frac{(S_0 - p)^2}{2(\sigma_S^2 + \sigma_u^2)} + \frac{1}{2}\ln\left(1 + \frac{\sigma_S^2}{\sigma_u^2}\right)}\right), \quad (6)$$

where γ^{-1} is the inverse function of γ, $\sigma_S^2 \equiv \frac{1}{\beta_S}$ and $\sigma_u^2 \equiv \frac{1}{\beta_u}$ are variances of the sentiment and unexplainable part of the fundamental.

Proof. See Appendix A.1. ■

Lemma 1 features the realistic property that retail investors only participate in trading risky assets if they are sufficiently wealthy. Furthermore, the condition (6) is helpful for interpreting the Covid-19 situation. First of all, it is natural to expect that the variance, σ_u^2, increased when the Covid-19 pandemic emerged: there was greater uncertainty about the value of firms and the

Similar market efficiency condition also features in models with traders who have market power such as Kyle (1985) and Holden and Subrahmanyam (1992). Many other papers build on these seminal settings.

economy in general. This effect alone would suggest that the threshold level of wealth needed for participating in the stock markets would have increased and led to less participation. This however is inconsistent with the stylized facts discussed in Section 2, which indicate that more investors participated, and arguably new less wealthy investors started to trade stock. Based on (6) there are two realistic effects that could explain the surge of new investors, by reducing the wealth threshold of participation. First, $\bar{w}$ is lower when the cost of participation, F, is lower. It is plausible that some non-monetary costs, like the opportunity cost of time, could have become smaller. This effect also suggests why new investors would have incentives to use trading apps and platforms that do not charge fixed fees. Second, a larger perceived wedge between the prices and the expected sentiment, $(S_0 - p)^2$ is another realistic driver of greater participation. [22] Finally, a higher perceived precision of the sentiment, β_S, (lower perceived variance) has an ambiguous effect on participation incentives. If the wedge $(S_0 - p)^2$ was small, a higher perceived precision (lower variance) would reduce participation. However, when the wedge is large enough, a higher perceived precision of S would further increase participation.

Given this participation threshold, we can derive the aggregate demand by retail investors as

$$H = N\int_i \mathbf{1}_i h_i di. = N\int_{\bar{w}}^{\infty} \gamma(\bar{w})\,\beta_u (S-p)\,dG(w_i) = N\beta_u (S-p)\,\gamma(\bar{w})\,(1 - G(\bar{w}))\,. \tag{7}$$

To shorten the notation, we will use $\bar{\gamma}(\bar{w}) \equiv \gamma(\bar{w})(1 - G(\bar{w}))$ to denote the average risk aversion of participating retail investors, which in turn is determined by the threshold $\bar{w}$, and the wealth distribution.

The analysis so far has kept the wealth distribution unchanged. As discussed in the introduction, it is also plausible that the Covid-19 shock had direct negative wealth effects. To capture this, consider that the pre-Covid-19 wealth distribution first order stochastically dominates the during-Covid-19 wealth distribution, i.e., $G_{before}(w_i) \leq G_{during}(w_i)$ for all w_i, with strict inequality for some w_i. This effect alone would have lowered the retail investors' demand, ceteris paribus, as it would imply that $1 - G_{during}(\bar{w}) \leq 1 - G_{before}(\bar{w})$, and for many distributions and values of $\bar{w}$ this inequality would be strict.

To summarize, a less favorable wealth distribution and a higher uncertainty ($\sigma_u^2 = 1/\beta_u$) would both reduce retail investors' participation. These effects are offset if the retail investors perceive a large wedge between the asset prices and their (possibly biased) expectations and the effective participation costs are lower. These considerations are necessary to explain the

[22] Both of these effects seem crucial for explaining the stylized facts discussed. It is worth noting that if retail investors were the be trading primarily based on fundamental information, we would have a similar participation condition with S_0 replaced by θ_0, and σ_S^2 replaced by σ_θ^2. However, it would be much more challenging to explain why unbiased prior beliefs about the fundamental value would be far from the prices. This together with empirical evidence of behavioural motives of online trading seems more consistent with a sentiment based explanation. See also Section XXX.

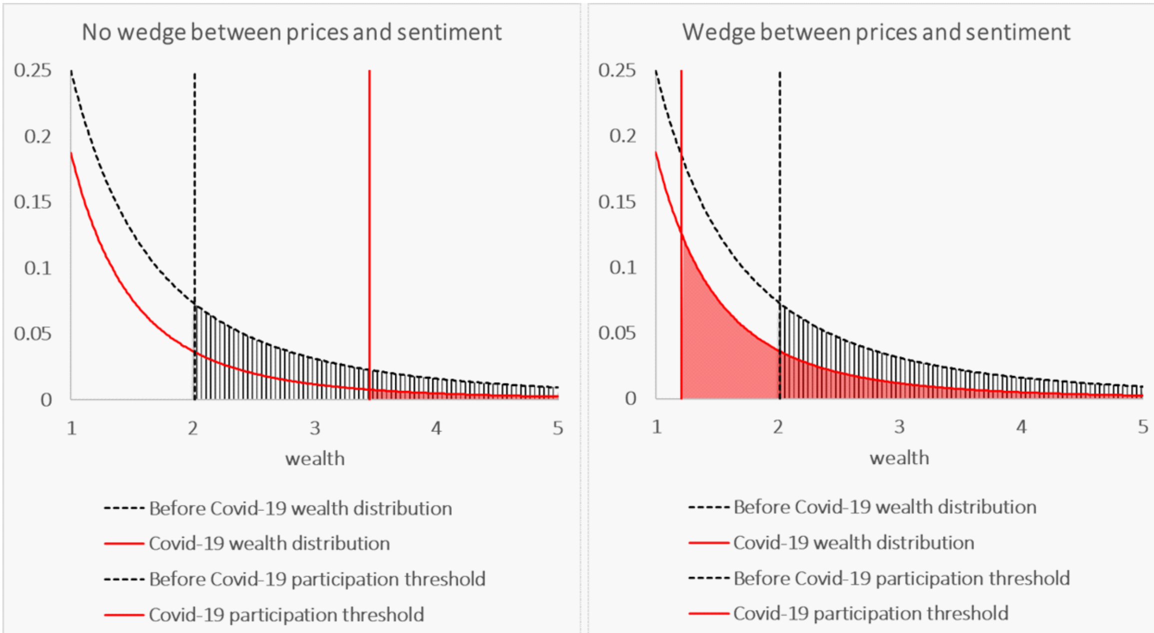

Figure 3: **Covid-19 impact and stock market participation.** These figures consider Pareto wealth distribution (originally applied and often used to describe the distribution of wealth in a society). The before Covid-19 distribution assumes scale parameter $w_m = 1$ and shape parameter $\alpha = 2$. Other parameters in the before Covid-19 economy are: $\gamma_c = 1$, $\sigma_u^2 = 1$, $\sigma_S^2 = 1, F = 0.7$, $S_0 = p$. The Covid scenarios assume that: 1) wealth distribution becomes less favourable in the sense of first order stochastic dominance, captured by $\alpha = 3$; 2) Uncertainty increasese, $\sigma_u^2 = 1$. The wedge $|S_0 - p| = 0$ on the left panel and $|S_0 - p| = 1.5$ on the right panel.

surge of new investors. Figure 3 illustrates these effects, by considering that risk tolerance is linearly increasing in wealth $\gamma(w_i) = \gamma_c w_i$, and by considering Pareto wealth distribution with scale parameter w_m and shape parameter $\alpha > 1$. On both figures, the black dotted lines give an illustrative example of pre-Covid-19 wealth distribution and threshold (6). The shaded area with vertical stripes represents the mass of stock market participants. The solid red lines cover the Covid-19 period, and on both panels of Figure 3, we assume that uncertainty increases (i.e., higher $\sigma_u^2 = 1/\beta_u$), and the pandemic leads to a wealth distribution that is stochastically dominated by the pre-crisis wealth distribution (higher parameter α). On the left panel, we consider that there is no wedge, i.e., $S_0 = p$, and on the right panel we consider a large enough positive wedge, $S > p$. The red area represents the mass of stock market participants. Whether stock market participation increases or decreases due to Covid-19 is determined by comparing the red and the striped area on each graph. We can see that a realistic explanation of increased stock market participation seems to require a large enough difference $|S_0 - p|$ to offset the negative wealth and uncertainty effects that would alone reduce participation.

3.3 Equilibrium asset prices

The aggregate demand by retail investors was specified in the previous Section, and following a similar derivation, the individual and aggregate demand by sophisticated informed investors are respectively

$$d = \tau\beta_u(\theta - p) \tag{8}$$

and $D = I\tau\beta_u(\theta - p)$. From this and (7) the total order flow/limit order schedule by these two investor groups is

$$H + D = N\beta_u(S - p)\bar{\gamma}(\bar{w}) + I\tau\beta_u(\theta - p)\,. \tag{9}$$

Competitive market makers observe the total demand, and use it to Bayesian update their beliefs about θ and the fundamental value v. Market makers do not consider S to be payoff relevant and it is a source uncertainty in the order flow.[23] Following a similar approach to Vives (1995), we can now solve for the equilibrium price.

Lemma 2 *The equilibrium price is*

$$p = (1 - \kappa_\theta)\theta_0 + \kappa_\theta\theta + \kappa_S S, \tag{10}$$

where

$$\begin{aligned} \kappa_\theta &\equiv \frac{(I\tau)^2\beta_S}{(N\bar{\gamma}(\bar{w}))^2\beta_\theta + (I\tau)^2\beta_S} \\ \kappa_S &\equiv \frac{N\bar{\gamma}(\bar{w})(I\tau)\beta_S}{(N\bar{\gamma}(\bar{w}))^2\beta_\theta + (I\tau)^2\beta_S}. \end{aligned} \tag{11}$$

Proof. See Appendix A.2. ■

Equilibrium price is a linear function of the prior mean of the fundamental, θ_0, the explainable component of the fundamental,θ, and retail investors' sentiment, S. We can see that the equilibrium price does not directly depend on the degree of uncertainty, $\sigma_u^2 = 1/\beta_u$, and it only depends on this parameter via its impact on retail investors' participation decisions and the average risk tolerance of $\bar{\gamma}(\bar{w})$ participating retail investors. This is because market makers are risk neural and retail and institutional investors have the same assessment of unexplainable part of the fundamental. This also implies that asset price volatility is affected by the degree of uncertainty $\sigma_u^2 = 1/\beta_u$ solely via retail investors' participation decisions.

We also see that an increased retail investor participation, which translates into a higher $\bar{\gamma}(\bar{w})$ reduces the relevance of information about the fundamental as the weight $\frac{\partial\kappa_\theta}{\partial\bar{\gamma}(\bar{w})} < 0$. At

[23]Noise in the order flow is necessary to avoid the Grossman and Stiglitz (1980) paradox. If all retail investors would be informed about θ instead of following the signal S and or the market makers would observe S, it would be necessary to introduce other sources of noise. Such modifications would not alter the key messages.

the same time it increases the impact of prior expectations θ_0. Interestingly, the impact of retail investor sentiment is non-monotonic in retail investor participation.

Corollary 3 *The impact of sentiment, S, on asset prices is maximized at an intermediate level of retail investor participation. Namely, weight κ_S is at its highest when $\bar{\gamma}(\bar{w}) = \gamma^* = \tau \frac{I}{N} \left(\frac{\beta_S}{\beta_\theta}\right)^{\frac{1}{2}}$*

Proof. This follows from $\gamma^* = \underset{\bar{\gamma}(\bar{w})}{\arg\max} \kappa_S$, where κ_S is defined in (11). The first order condition implies γ^*, and the second order condition confirms it as maximum. ■

When there are few retail investors, their total demand, which depends on S, is small and does not have that much impact on prices. At the same time, when there are many retail investors, their total order flow becomes very noisy and less informative about the fundamental value. Namely, the variance of the signal that the market makers obtain from the order flow is $Var(z|\theta) = \frac{(N\bar{\gamma}(\bar{w}))^2}{(I\tau)^2} \frac{1}{\beta_S}$ increasing in $\bar{\gamma}(\bar{w})$ (see Appendix A.2). Consequently, the market makers set the price close to the prior mean θ_0 and both the sentiment and the informed traders' information has less impact. The value of $\bar{\gamma}(\bar{w}) = \gamma^*$ that maximizes the weight on the sentiment is proportional to the informed investors' risk tolerance and higher if retail investors believe that their signal is more precise, β_S. A higher precision of fundamental information, β_θ, reduces γ^*.

As argued in the precious section, large enough shocks to the wealth distribution, uncertainty in the economy, and the wedge between prices and sentiment, can all lead to fast changes in the retail investors' participation in trading, which this section shows can lead to fast price changes and changes in the sensitivity of prices to different sources of uncertainty.

As more standard results from a comprative statics standpoint, if the prior mean θ_0 is more precise (higher β_θ) then the prices are closer to the prior mean and information shocks, θ and S, have a lesser impact. At the same time, when the retail investors consider their signal S to be more precise, the weight of the prior mean is relatively lower. It is also intuitive that when informed investors are more risk tolerant (higher τ), they trade more in the equilibrium and their informative signal θ has a greater impact on equilibrium prices.

Finally, we can derive the unconditional variance of the risk asset price, given information that is available at date 0.[24] From (10) and (11) we obtain that

$$Var(p) = \frac{\kappa_\theta^2}{\beta_\theta} + \frac{\kappa_S^2}{\beta_S} = \frac{1}{\left(\frac{N\bar{\gamma}(\bar{w})}{I\tau}\right)^2 \frac{\beta_\theta}{\beta_S} + 1} \cdot \frac{1}{\beta_\theta}$$

We can see that even though a greater retail investor participation has complex effect on different drivers of asset prices, it always reduces the unconditional volatility of prices by making

[24]Note that $\bar{\gamma}(\bar{w})$ is known at this point as it depends on publicly observable variables.

the market more liquid. The effect of other parameters is also intuitive: a higher precision of informed signals, β_θ, reduces the variance of the price and if retail investors perceive the sentiment signal, β_S, to be more precise, the prices are more volatile.

4 Extension: presence of informed retail investors.

The assumption in the baseline model is that all retail investors are trading based on a sentiment signal, and do not have fundamental information. Consequently, there traders make losses in expectations. While the idea that retail investors tend to make losses on average is consistent with a number of empirical findings discussed in the introduction, this assumption may sound strong. Certainly some retail investors could have fundamental information and profit from trading on it.

To allow this possibility, we extend the setting as follows. A fraction $\lambda \in [0,1)$ of retail investors are informed and observe θ. The probability of being informed is independent of the investor's wealth. If an informed retail investor participates, the optimal demand schedule h_i^I must maximize

$$U_i^I\left(\mathbf{1}_i^I=1\right) \equiv \mathbb{E}\left[-\exp\left(-\frac{h_i^I(\theta-p)+w_i-F}{\gamma(w_i)}\right)|\theta,w_i\right].$$

The utility from non-participation remains the same, and an informed retail investor i participates, if and only if, $\mathbb{E}\left[U_i^I\left(\mathbf{1}_i^I=1\right)\right] \geq U_i(\mathbf{1}_i=0)$. The remaining fraction $(1-\lambda)$ of retail investors solve the same problem as in in the main setting. The wealth threshold for participation may now differ among retail investors, and we denote the demand of informed retail investors and the demand of sentiment following retail investors with H^I and H^S, respectively. The total demand of retail investors is $H = H^I + H^S$. The informed institutional investors' problem remains unchanged and the market makers take the new structure into account.

The solution method of this problem follows the same steps as in Section 3. Naturally, the only difference in the informed retail trader's problem compared to earlier is that his/her decision is based on the distribution of θ, rather than S. There are now two, potentially different wealth levels, at which different types of retail investors participate These are

$$\bar{w}^{I} = \gamma^{-1}\left(\frac{F}{\frac{(\theta_0-p)^2}{2\left(\sigma_\theta^2+\sigma_u^2\right)}+\frac{1}{2}\ln\left(1+\frac{\sigma_\theta^2}{\sigma_u^2}\right)}\right)$$

$$\bar{w} = \gamma^{-1}\left(\frac{F}{\frac{(S_0-p)^2}{2\left(\sigma_S^2+\sigma_u^2\right)}+\frac{1}{2}\ln\left(1+\frac{\sigma_S^2}{\sigma_u^2}\right)}\right),$$

where informed retail investors participate as long as their wealth is at least $\bar{w}^{I}$, and the rest participates as long as their wealth is at least $\bar{w}$, as before. In stable times, these thresholds may be similar or may be ranked either way. However, as it is arguably plausible that the wedge between the price and the expected sentiment, $|S_0 - p|$, during Covid-19 crisis was higher than the wedge between the price and prior beliefs, $|\theta_0 - p|$, it seems more likely that $\bar{w}$ was lower than $\bar{w}^{I}$, and new investors who started to participate were more often retail investors who rely on sentiment rather than on unbiased fundamental information. Furthermore, it is hard to find compelling justification why a wedge $|\theta_0 - p|$ would remain persistently high, as in many rational settings, prior beliefs often reflect information revealed by past prices. As shown in Section 3.2, such a wedge would need to be there to justify the surge in new retail investors during Covid-19.

The derivation of aggregate demand and equilibrium prices can nevertheless maintain flexibility regarding the comparison of $\bar{w}^{I}$ and $\bar{w}$. Provided these threshold, we derive that $H^I = N\lambda\beta_u\left(\theta-p\right)\bar{\gamma}\left(\bar{w}^I\right)$ and $H^S = N\left(1-\lambda\right)\beta_u\left(S-p\right)\bar{\gamma}\left(\bar{w}\right)$, where $\bar{\gamma}\left(\bar{w}\right)=\gamma\left(\bar{w}\right)\left(1-G\left(\bar{w}\right)\right)$ and $\bar{\gamma}\left(\bar{w}^S\right)=\gamma\left(\bar{w}^S\right)\left(1-G\left(\bar{w}^S\right)\right)$.

We can then derive the equilibrium price[25] in this setting as

$$p = \left(1-\kappa_\theta\right)\theta_0 + \kappa_\theta\theta + \kappa_S S,$$

where

$$\kappa_\theta = \frac{\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right)+I\tau\right)^2\beta_S}{\left(N\bar{\gamma}\left(\bar{w}\right)\right)^2\beta_\theta+\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right)+I\tau\right)^2\beta_S}$$

$$\kappa_S = N\left(1-\lambda\right)\frac{\bar{\gamma}\left(\bar{w}\right)\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right)+I\tau\right)\beta_S}{\left(N\bar{\gamma}\left(\bar{w}\right)\right)^2\beta_\theta+\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right)+I\tau\right)^2\beta_S}.$$

Comparing this with (10) and (11) we can see that all the main results hold and the weights on components that determine the equilibrium price have slightly changed. Namely, when a

[25]See details in Appendix A.3.

larger proportion of retail investors are informed, the equilibrium price has a higher weight on the signal about the fundamental, θ. This is a natural outcome of there being more informed traders and thus the variance of the signal that the market makers obtain from the order flow is now $Var\left(z|\theta\right)=\frac{\left(N\bar{\gamma}(\bar{w})\right)^2}{\left(N\lambda\bar{\gamma}(\bar{w}^I)+I\tau\right)^2}\frac{1}{\beta_S}$, which is decreasing in λ (see Appendix A.3).

Overall all highlighted effects of the main model remain qualitatively the same, while the impact of sentiment on the overall demand by retail investors and the effects of sentiment and the prior on prices are smaller in magnitude.

5 Final remarks

The paper documented some stylized patterns of stock trading by retail investors during the year 2020. It also provided a REE model to highlight the drivers of these patterns and derived new results on the relationship between stock market participation and sentiment analysis. The incoming of retail traders who coordinate on sentiment rather than on fundamental information, is crucial for explaining these recent patterns in th econtext of this model.

A Proofs

A.1 Proof of Lemma 1

Using $\mathbb{E}[c_i|S] = h_i(S-p) + w_i - F$, $Var[c_i|S] = h_i^2 \frac{1}{\beta_u}$, and the optimal demand (5) in (4), we obtain

$$U_i(\mathbf{1}_i = 1) = -\exp\left(-\frac{1}{\gamma(w_i)}(w_i - F)\right)\exp\left(-\frac{\beta_u(S-p)^2}{2}\right)$$

When the retail investors decide whether to participate, they know their wealth and price, but do not know S. It follows that

$$\mathbb{E}[U_i(\mathbf{1}_i = 1)] = -\exp\left(-\frac{1}{\gamma(w_i)}(w_i - F)\right)\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi/\beta_S}}\exp\left(-\frac{\beta_u(S-p)^2}{2}\right)\exp\left(-\frac{\beta_S(S-S_0)^2}{2}\right)dS$$

After simplifying, we obtain

$$\mathbb{E}[U_i(\mathbf{1}_i = 1)] = -\exp\left(-\frac{1}{\gamma(w_i)}(w_i - F) - \frac{\beta_u\beta_S(S_0-p)^2}{2(\beta_u+\beta_S)}\right)\left(\frac{\beta_S}{(\beta_u+\beta_S)}\right)^{\frac{1}{2}}$$

It then follows that $\mathbb{E}[U_i(\mathbf{1}_i = 1)] \geq U_i(\mathbf{1}_i = 0)$ if and only if

$$\gamma(w_i) \geq \frac{F}{\frac{(S_0-p)^2}{2\left(\frac{1}{\beta_S}+\frac{1}{\beta_u}\right)} + \frac{1}{2}\ln\left(1+\frac{\beta_u}{\beta_S}\right)}$$

As $\gamma(w_i)$ is monotonically increasing in w_i, we obtain that investor i participates, if and only if,

$$w_i \geq \bar{w} \equiv \gamma^{-1}\left(\frac{F}{\frac{(S_0-p)^2}{2\left(\frac{1}{\beta_S}+\frac{1}{\beta_u}\right)} + \frac{1}{2}\ln\left(1+\frac{\beta_u}{\beta_S}\right)}\right).$$

A.2 Proof of Lemma 2

From (9) the total limit order schedule is

$$L(p) = H + D = N\beta_u\bar{\gamma}(\bar{w})S + I\tau\beta_u\theta - \beta_u(N\bar{\gamma}(\bar{w}) + I\tau)p. \qquad (12)$$

The signal that the market makers observe from this is

$$z \equiv \frac{L(p) + \beta_u(N\bar{\gamma}(\bar{w}) + I\tau)p}{I\tau\beta_u} = \theta + S\frac{N\bar{\gamma}(\bar{w})}{I\tau},$$

and it follows that $z|\theta \sim \mathcal{N}\left(\theta, \frac{(N\bar{\gamma}(\bar{w}))^2}{(I\tau)^2}\frac{1}{\beta_S}\right)$. By Bayes's Rule and properties of the normal distribution, it then follows that the conditional distribution

$$v|z = \theta + u|z \sim \mathcal{N}\left(\frac{\beta_\theta\theta_0 + \beta_S\frac{(I\tau)^2}{(N\bar{\gamma}(\bar{w}))^2}z}{\beta_\theta + \beta_S\frac{(I\tau)^2}{(N\bar{\gamma}(\bar{w}))^2}}, \frac{1}{\beta_\theta + \beta_S\frac{(I\tau)^2}{(N\bar{\gamma}(\bar{w}))^2}} + \frac{1}{\beta_u}\right)$$

Setting the price equal to the fundamental value based of all public information

$$p = E\left[v|H + D\right] = E\left[v|z\right] = \frac{\beta_\theta\theta_0 + \beta_S\frac{(I\tau)^2}{(N\bar{\gamma}(\bar{w}))^2}\left(\frac{L(p)+\beta_u(N\bar{\gamma}(\bar{w})+I\tau)p}{I\tau\beta_u}\right)}{\beta_\theta + \beta_S\frac{(I\tau)^2}{(N\bar{\gamma}(\bar{w}))^2}}$$

and rearranging

$$L\left(p\right) = p\left(\frac{\left(N\bar{\gamma}\left(\bar{w}\right)\right)^2}{I\tau}\frac{\beta_\theta}{\beta_S} - N\bar{\gamma}\left(\bar{w}\right)\right)\beta_u - \frac{\left(N\bar{\gamma}\left(\bar{w}\right)\right)^2}{I\tau}\frac{\beta_\theta}{\beta_S}\beta_u\theta_0$$

Equating this with (12), we obtain the equilibrium price as

$$p = \frac{\left(N\bar{\gamma}\left(\bar{w}\right)\right)^2\beta_\theta}{\left(N\bar{\gamma}\left(\bar{w}\right)\right)^2\beta_\theta + \left(I\tau\right)^2\beta_S}\beta_\theta\theta_0 + \frac{\left(I\tau\right)^2\beta_S}{\left(N\bar{\gamma}\left(\bar{w}\right)\right)^2\beta_\theta + \left(I\tau\right)^2\beta_S}\theta + \frac{N\bar{\gamma}\left(\bar{w}\right)I\tau\beta_S}{\left(N\bar{\gamma}\left(\bar{w}\right)\right)^2\beta_\theta + \left(I\tau\right)^2\beta_S}S$$

Defining the coefficients κ_θ and κ_S, then proves Lemma 2.

A.3 Derivation of equilibrium price in a setting where some retail investors are informed

The total limit order schedule is now $H^I = N\lambda\beta_u\left(\theta - p\right)\bar{\gamma}\left(\bar{w}^I\right)$ and $H^S = N\left(1-\lambda\right)\beta_u\left(S - p\right)\bar{\gamma}\left(\bar{w}\right)$

$$L\left(p\right) = H + D = H^I + H^S + D = N\lambda\beta_u\left(\theta - p\right)\bar{\gamma}\left(\bar{w}^I\right) + N\left(1-\lambda\right)\beta_u\left(S - p\right)\bar{\gamma}\left(\bar{w}\right) + I\tau\beta_u\left(\theta - p\right)$$

The signal that the market makers observe from this is

$$z \equiv \frac{L\left(p\right) + \beta_u\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right) + N\left(1-\lambda\right)\bar{\gamma}\left(\bar{w}\right) + I\tau\right)p}{\beta_u\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right) + I\tau\right)} = \theta + S\frac{N\bar{\gamma}\left(\bar{w}\right)}{\beta_u\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right) + I\tau\right)},$$

and $z|\theta \sim \mathcal{N}\left(\theta, \frac{(N\bar{\gamma}(\bar{w}))^2}{(N\lambda\bar{\gamma}(\bar{w}^I)+I\tau)^2}\frac{1}{\beta_S}\right)$. The market makers set the price

$$p = E\left[v|z\right] = \frac{\beta_\theta \theta_0 + \beta_S \frac{\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right)+I\tau\right)^2}{(N\bar{\gamma}(\bar{w}))^2}\left(\frac{L(p)+\beta_u\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right)+N(1-\lambda)\bar{\gamma}(\bar{w})+I\tau\right)p}{\beta_u(N\lambda\bar{\gamma}(\bar{w}^I)+I\tau)}\right)}{\beta_\theta + \beta_S \frac{(N\lambda\bar{\gamma}(\bar{w}^I)+I\tau)^2}{(N\bar{\gamma}(\bar{w}))^2}}$$

Following the same steps as in the Proof of 2, we obtain that the equilibrium price is

$$p = \frac{\beta_\theta \left(N\bar{\gamma}\left(\bar{w}\right)\right)^2}{\beta_\theta \left(N\bar{\gamma}\left(\bar{w}\right)\right)^2 + \beta_S \left(N\lambda\bar{\gamma}\left(\bar{w}^I\right) + I\tau\right)^2}\theta_0 + \frac{\beta_S \left(N\lambda\bar{\gamma}\left(\bar{w}^I\right) + I\tau\right)^2}{\beta_\theta \left(N\bar{\gamma}\left(\bar{w}\right)\right)^2 + \beta_S \left(N\lambda\bar{\gamma}\left(\bar{w}^I\right) + I\tau\right)^2}\theta$$
$$+ N\left(1-\lambda\right)\frac{\bar{\gamma}\left(\bar{w}\right)\beta_S\left(N\lambda\bar{\gamma}\left(\bar{w}^I\right) + I\tau\right)}{\beta_\theta \left(N\bar{\gamma}\left(\bar{w}\right)\right)^2 + \beta_S \left(N\lambda\bar{\gamma}\left(\bar{w}^I\right) + I\tau\right)^2}S$$

B Canadian platforms for online stock trading

Table B.1 Traditional Banks Offering Online Trading Services

Name	Year established/ notable acquisitions	Account Minimum	Commission per equity trade	Apple App Store rating
BMO InvestorLine	1988	$15,000	$9.95	1.6/5
CIBC Investor's Edge	Key acquisitions in the brokerage business: Wood, Gundy & Co (1988); Merrill Lynch & Company's retail brokerage business (2001)	$10,000	$4.95 -$6.95	3.3/5
RBC Direct Investing	Brokerage division of Royal Bank of of Canada (1860)	$15,000	$6.95-$9.95	4.8/5
Scotia iTRADE	Acquired TradeFreedom Securities and ETrade (2007)	$10,000	$4.99	1.2/5
TD Direct Investing	2012 TD Green Line Investor Services (1984)	$15,000	$7-$9.99	4.5/5

Table B.2 Specialized Online Trading Platforms

Name	Year established	Account Minimum	Commission per equity trade	Apple App Store rating
Interactive Brokers	1978	US $10,000 (or non-USD equivalent) - first 8 months	Tiered vs Fixed	2.6/5
Questrade	1999	$1000	1¢ per share ($6.95 max)	1.3/5
Qtrade Investor	2000	-	$6.95 for equity trades	2.2/5
Wealthsimple Trade	2014	0	0	4.7/5
Virtual Brokers	2009	$5000	0	2.5/5

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